

Please check that this question paper contains 9 questions and 2 printed pages within first ten minutes.

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Uni. Roll No.

Program/ Course: B.Tech.

Semester : 4

Name of Subject: Mathematics-III

Subject Code: BSME-101

Paper ID: 16197

Time Allowed: 03 Hours

Max. Marks: 60

NOTE:

- 1) Part – A & B are compulsory
- 2) Part- C has two Questions Q8 & Q9 and both are compulsory, but with internal choice.
- 3) Any missing data may be assumed appropriately.

Part – A

[Marks: 02 each]

Q1.

- a) Write one dimensional wave equation.
- b) Write Cauchy – Riemann equations in polar co-ordinates for analytic function
 $f(z) = u + i v$
- c) Classify the following partial differential equation:

$$x^2 \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + u = 0$$
- d) A normal population has a mean of 6.8 and standard deviation of 1.5 . A sample of 400 members gave a mean 6.75. Is the difference significant? $z_\alpha = 1.96$ at 5% level of significance.
- e) Evaluate $\oint_c \frac{3z^2+z}{z^2-1}$, where c is the circle $|z| = \frac{1}{2}$.
- f) Find the mean of the Binomial Distribution.

Part – B

[Marks: 04 each]

Q2. Solve the following partial differential equation:

$$\frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = x^2 + xy + y^2$$

Q3. Solve the following partial differential equation:

$$(y - z) \frac{\partial z}{\partial x} + (x - y) \frac{\partial z}{\partial y} = z - x$$

Q4. Assume that on the average one telephone number out of fifteen called between 2 P.M and 3 P.M on week-days is busy. What is the probability that if 6 randomly selected telephone numbers are called (i) not more than three, (ii) at least three of them will be busy.

- Q5. If $f(z) = u + iv$ is an analytic function, then prove that u and v are both harmonic functions.
- Q6. Expand the function $\frac{1}{(z+1)(z+3)}$ in the region $1 < |z| < 3$.
- Q7. Find the correlation coefficient between x and y , when the lines of regression are :
 $2x - 9y + 6 = 0, \quad x - 2y + 1 = 0$

Part - C

[Marks: 12 each]

- Q8. A rectangular plate with insulated surfaces is 8 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature along one short edge $y = 0$ is given by $u(x, 0) = 100 \sin \frac{\pi x}{8}$, $0 < x < 8$, while the two long edges $x = 0$ and $x = 8$ as well as the other short edge are kept at 0°C , show that the steady state temperature at any point of the plate is given by
 $u(x, y) = 100 e^{-\frac{\pi y}{8}} \sin \frac{\pi x}{8}$.

OR

If the probability that an individual suffers a bad reaction from a certain injection is 0.001. Find the probability that out of 2000 individuals

- (i) Exactly 3 individuals will suffer a bad reaction
 (ii) None will suffer a bad reaction
 (iii) More than one individual will suffer
 (iv) More than two individuals will suffer.
- Q9. Evaluate $\int_0^\pi \frac{d\theta}{a+b\cos\theta}$, where $a > |b|$, using Residue theorem.
 Hence evaluate $\int_0^{2\pi} \frac{d\theta}{\sqrt{2-\cos\theta}}$.

OR

A die is thrown 276 times and the result of these thrown are given below :

No. appeared on die	1	2	3	4	5	6
Frequency	40	32	29	59	57	59

Test whether the die is biased or not. Tabulated value of chi square at 5% level of significance for 5d.f is 11.09.
