

Please check that this question paper contains 9 questions and 2 printed pages within first ten minutes.

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Uni. Roll No. ....

Program: B.Tech. (Batch 2018 onward)

Semester: 4

Name of Subject: Mathematics-III

Subject Code: BSCE-101

Paper ID: 16180

Time Allowed: 03 Hours

Max. Marks: 60

NOTE:

- 1) Parts A and B are compulsory
- 2) Part-C has Two Questions Q8 and Q9. Both are compulsory, but with internal choice
- 3) Any missing data may be assumed appropriately

**Part – A**

[Marks: 02 each]

Q1.

- a) State Fourier Integral Theorem.
- b) Find  $L^{-1}\left(\frac{1}{p(p-3)}\right)$ .
- c) Define partially ordered set.
- d) State and prove first shifting property of finding Laplace transform.
- e) Consider the group  $(Z,+)$  and let  $H = \{3n : n \in Z\}$ . Show that  $H$  is a subgroup of  $Z$ .
- f) Prove that every distributive lattice is modular.

**Part – B**

[Marks: 04 each]

Q2. Find the Laplace transform of  $\frac{1 - \cos 2t}{t}$ .

Q3. Find the Fourier transform of  $f(x) = \begin{cases} 1-x^2 & \text{if } |x| \leq 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ .

Q4. Find the right cosets of the subgroup  $\{1,-1\}$  of the group  $\{1,-1,i,-i\}$  under multiplication.

Q5. Show that the set  $G = \{1,2,3,4,5,6\}$  is a group under multiplication modulo 7.

- Q6. Using Parseval's identity, prove that  $\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2ab(a+b)}$ .
- Q7. Show that for a bounded, distributive lattice, complement of an element is unique.

## Part – C

[Marks: 12 each]

- Q8. Using Laplace transform, solve the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t} \sin t, \text{ where } x(0) = 0 \text{ and } x'(0) = 1.$$

OR

Use Fourier sine transform to solve the equation  $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$  under the conditions

$$u(0,t) = 0, u(x,0) = e^{-x} (x > 0), u(x,t) \text{ is bounded where } x > 0, t > 0.$$

- Q9. (a) State and prove Lagrange's Theorem.  
 (b) Prove that intersection of two normal subgroups of a group is a normal subgroup.

OR

Prove that product of two lattices is a lattice.

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